Observational motivation

Useful observations:

(1) The Sun. Accurate knowledge of mass, radius, luminosity, age (from isotope ratios in rocks). Observations of nonradial oscillations (helioseismology) $\rightarrow$ inference of internal structure. Information on nuclear reactions from neutrinos.

Helioseismology website: `http://www.gong.noao.edu/`

(2) Binaries. Most stars are binaries (see e.g. Duquennoy & Mayor 1991, A&A, 248, 485). For eclipsing systems (inclination $i \approx 90°$) where the radial velocity of the stars can be measured spectroscopically can obtain R_1, R_2, M_1 and M_2.

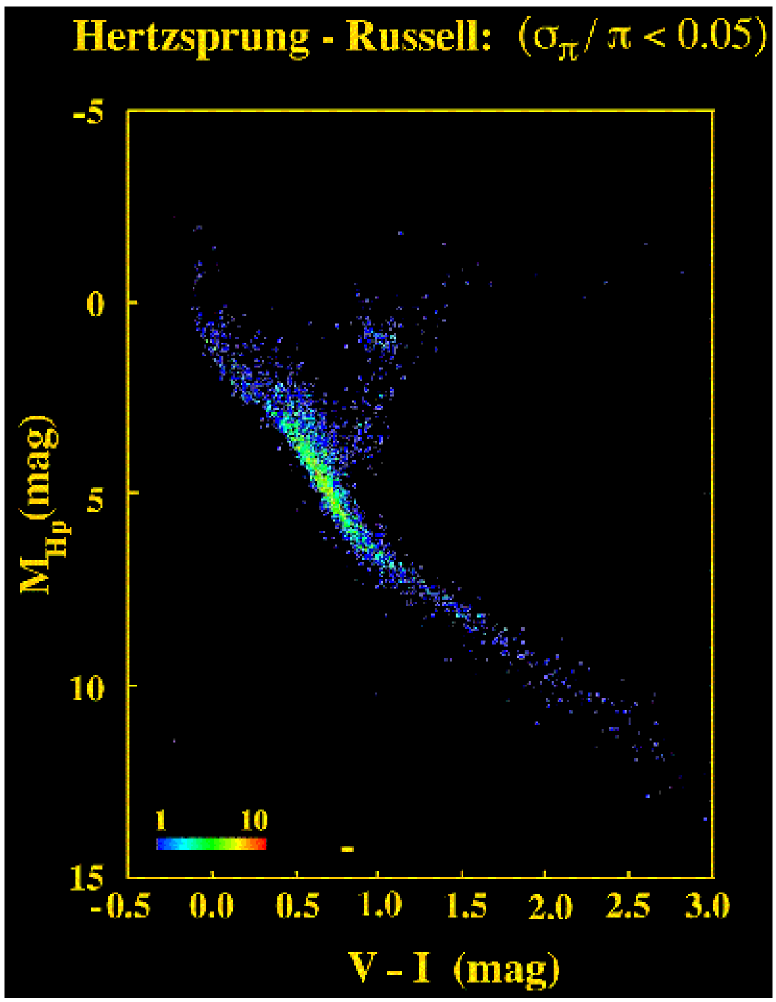
(3) Stars with known distances from parallaxes $\rightarrow$ known absolute magnitudes.

(4) Clusters. Open clusters in the disk and globular clusters (old, halo) provide sample of stars at a common distance. Same composition, same age.

Hertzsprung-Russell diagram

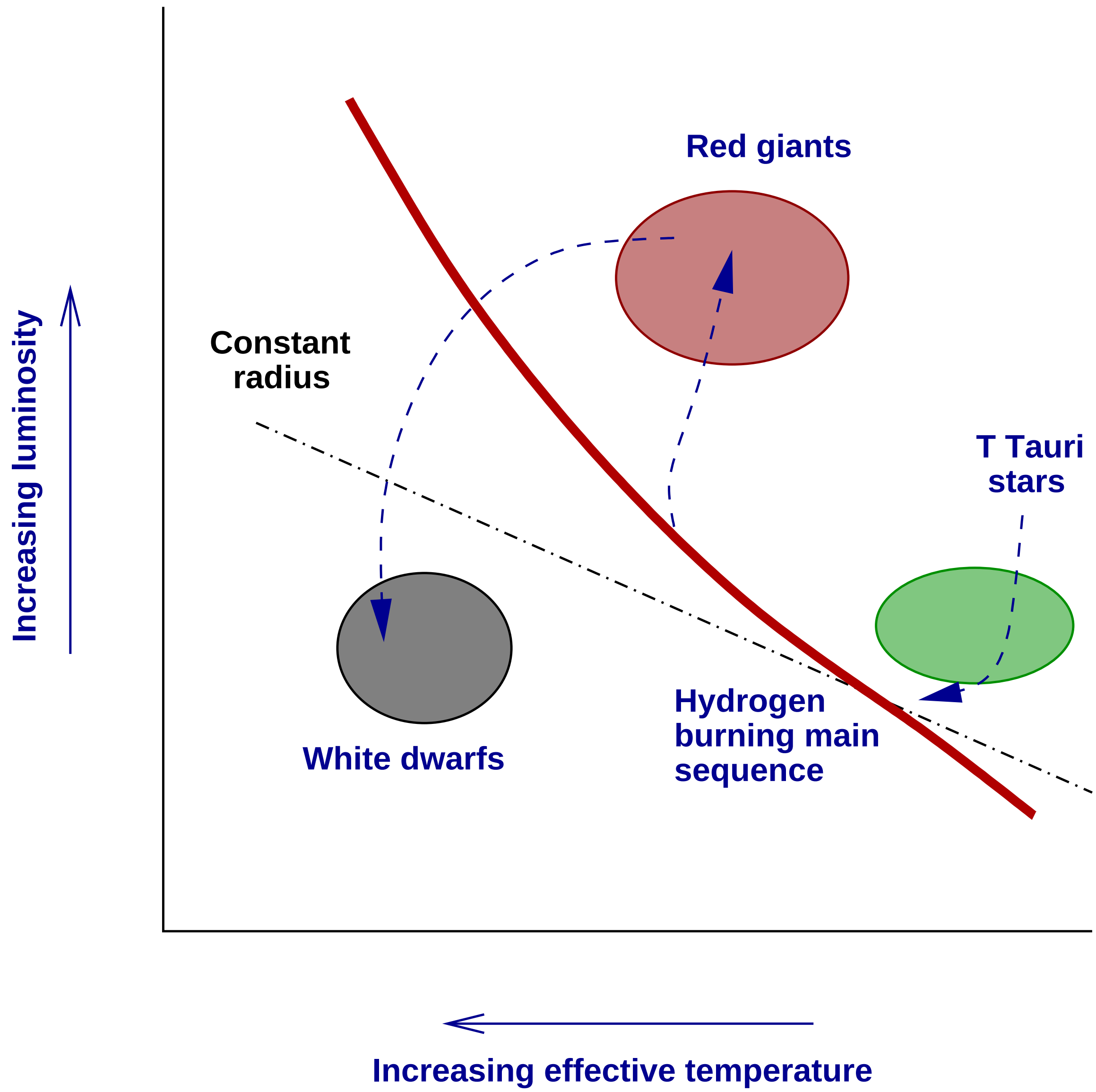
Plot of L vs T_e (theory) or M_V vs color or spectral type (observational). Stars do not randomly populate this diagram – observe main sequence, populations of pre-main-sequence and evolved stars, stellar remnants (white dwarfs).

e.g. stars with accurate distances measured by Hipparcos:

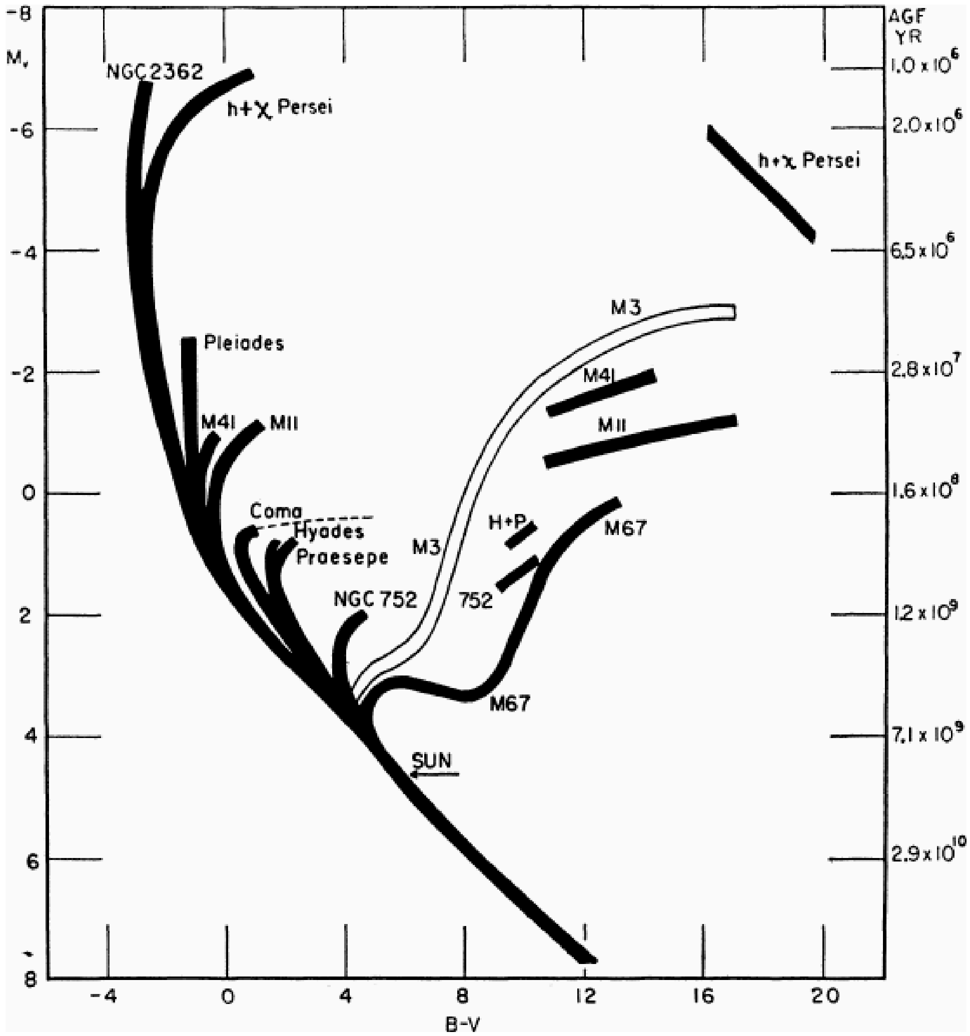


Mostly main sequence stars, some giants.

Theorist's HR diagram



Evolution with age



Rapid evolution of massive stars (hot, luminous) → earlier departure from main sequence. Fitting for the turn off point (and other features) yields age estimate.

Stellar Populations

Primordial nucleosynthesis yielded H, He + very small amounts of heavier elements ('metals'). Subsequent enrichment of the ISM from supernovae.

Observationally, observe two distinct populations in the Galaxy:

	Population I	**Population II**
Distribution	disk	halo / spheroid
	$\|z\| <\sim 200$ pc (O stars)	
Kinematics	disk rotation	$\sim$ no rotation
Radial dispersion	$v \sim 30$ km s^{-1}	large
Metallicity	$Z \simeq 0.02$	$Z < 0.01$
Age	young	old

i.e. consistent with expectations.

Metallicity has a strong influence on both the equation of state and the opacity. Spectrum and evolution of stars from different populations are thus different.

The virial theorem

Equations of stellar structure are local. Virial theorem is an integral theorem useful for interpreting stellar evolution.

Simplest derivation

Assume a static star with negligible bulk motion. Hydrostatic equilibrium gives:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho\frac{Gm}{r^2}$$

multiply by $V(r)dr = 4/3\pi r^3 dr$

$$VdP = -\frac{4}{3}\pi r^2 \rho dr \frac{Gm}{r}$$

$$VdP = -\frac{1}{3}\frac{Gm}{r}dm$$

integrate over star: LHS gives by parts,

$$\int VdP = PV|_0^R - \int PdV$$

$V = 0$ at $r = 0$. Taking $P = 0$ at $r = R$ then surface term vanishes. Identify RHS with gravitational binding energy of the star Ω,

$$d\Omega = -\frac{Gm}{r}dm$$

$$\Omega = -\int \frac{Gm}{r}dm$$

Result:

$$0 = \Omega + 3 \int P dV$$

For an equation of state:

$$P = (\gamma - 1)\rho E$$

where E is the internal energy per unit mass,

$$\int P dV = \int (\gamma - 1)\rho E dV$$

which for constant γ yields,

$$0 = \Omega + 3(\gamma - 1)U$$

with U the total internal energy of the star.

A better derivation

Can also derive the virial theorem with less restrictive assumptions (see HK1.3). Consider the equation of motion of a mass element dm at position $\mathbf{r}(t)$,

$$\ddot{\mathbf{r}} = -\frac{1}{\rho}\nabla P + \mathbf{F}$$

where P is the isotropic pressure (i.e. $P\delta_{ij}$), F are the forces acting on the element.

For scalar virial theorem take dot product with $\mathbf{r}$, integrate, and note that $dm = \rho dV$:

$$\int_m \mathbf{r} \cdot \ddot{\mathbf{r}} dm = -\int_V \mathbf{r} \cdot \nabla P dV + \int_m \mathbf{r} \cdot \mathbf{F} dm$$

Strategy:

- Get something related to K.E. from LHS
- First term on RHS $\rightarrow$ internal energy as before
- F is due to gravity $\rightarrow \Omega$

LHS:

$$\begin{aligned}
\mathbf{r} \cdot \ddot{\mathbf{r}} &= \frac{d}{dt}(\mathbf{r} \cdot \dot{\mathbf{r}}) - \dot{\mathbf{r}}^2 \\
&= \frac{1}{2}\frac{d^2}{dt^2}(\mathbf{r} \cdot \mathbf{r}) - \dot{\mathbf{r}}^2
\end{aligned}$$

Hence,

$$\int_m \mathbf{r} \cdot \ddot{\mathbf{r}} dm = \frac{1}{2}\ddot{I} - 2T$$

where we have defined:

$$I = \int_m r^2 dm$$

which is the moment of inertia, and,

$$T = \int_m \frac{1}{2}\dot{r}^2 dm$$

the internal bulk K.E. – e.g. rotation, turbulence.

RHS term 1:

$$\mathbf{r} \cdot \nabla P = \nabla \cdot (P\mathbf{r}) - 3P$$

$$\begin{aligned} -\int_V \mathbf{r} \cdot \nabla P dV &= -\int_V \nabla \cdot (P\mathbf{r}) dV + \int_V 3P dV \\ &= -\int_S (P\mathbf{r}) \cdot d\mathbf{S} + \int_V 3P dV \end{aligned}$$

If we choose a surface such that $P = P_s$ is a constant, then,

$$-\int_S (P_s \mathbf{r}) \cdot d\mathbf{S} = -P_s \int_s \mathbf{r} \cdot d\mathbf{S} = -P_s \int_V \nabla \cdot \mathbf{r} dV = -3P_s V$$

$P_s = 0$ is a special case.

As before, for an EOS,

$$P = (\gamma - 1)\rho E$$

and a constant γ,

$$\int_V 3P dV = 3(\gamma - 1)U$$

RHS term 2 is called the *virial of Clausius*. Suppose that the only force is due to self-gravity,

$$\mathbf{F} = -\nabla\phi$$

where

$$\phi(r) = -G\int\frac{dm'}{|\mathbf{r}-\mathbf{r}'|}$$

Define the gravitational self-energy – energy required to assemble star from infinity:

$$\Omega = -\frac{1}{2}G\int\int\frac{dmdm'}{|\mathbf{r}-\mathbf{r}'|}$$

(no factor of 1/2 in HK but their sums are only for $i < j$).

$$\begin{aligned}
\int\mathbf{r}\cdot\mathbf{F}dm &= -\int\mathbf{r}\cdot\nabla\phi dm \\
&= G\int\int(\mathbf{r}\cdot\nabla)\frac{dmdm'}{|\mathbf{r}-\mathbf{r}'|} \\
&= \frac{1}{2}G\int\int\left[\mathbf{r}\cdot\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}+\mathbf{r}'\cdot\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right]dmdm' \\
&= -\frac{1}{2}G\int\int\left[\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}+\frac{\mathbf{r}'\cdot(\mathbf{r}'-\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|^3}\right]dmdm' \\
&= \Omega
\end{aligned}$$

Finally we have:

$$\frac{1}{2}\ddot{I} = 2T + 3(\gamma - 1)U - 3P_sV + \Omega$$

which reduces to the previous result if the system is static, $\ddot{I} = 0$, has no internal motions, $T = 0$, and if we consider a zero pressure boundary, $P_s = 0$.

First problem set next week. Check:

`http://arXiv.org/archive/astro-ph`

for interesting preprints – e.g. today 'The Frequency Content of the VIRGO/SoHO Lightcurves: Implications for Planetary Transit Detection from Space'.