Consequences of the Eddington limit

Massive stars approach the Eddington limit due to the high luminosity from nuclear burning. Because accretion can be much more efficient than nuclear reactions at generating energy, Eddington limit most often relevant for accreting compact objects.

(1) Accreting compact objects:

Luminosity due to accretion at rate $\dot{M}$ is,

$$L = \frac{GM\dot{M}}{R}$$

which implies that the Eddington limiting luminosity is reached at an accretion rate,

$$\dot{M} = \frac{4\pi c m_H}{\sigma_T} R \approx 1.5 \times 10^{-8} \left(\frac{R}{10 \text{ km}} \right) M_\odot \text{ yr}^{-1}$$

...taking a radius appropriate for a neutron star and ignoring relativistic corrections. Mass transfer in a binary system can exceed this value.

(2) Accreting black holes:

Stable circular orbits around a black hole only exist outside the radius of marginal stability. For a non-rotating black hole:

$$r_{\rm ms} = \frac{6GM}{c^2}$$

while for a maximally rotating hole,

$$r_{\rm ms} = \frac{GM}{c^2}$$

for orbits rotating in the same sense as the hole and,

$$r_{\rm ms} = \frac{9GM}{c^2}$$

for retrograde orbits. If gas spirals in towards the hole through a sequence of circular orbits, and then plunges from $r_{\rm ms}$ across the horizon without emitting any further energy (or exerting any torque on the disk), then the efficiency of accretion is,

$$\begin{aligned} \epsilon \equiv \frac{L}{\dot{M}c^2} &= 0.057 \quad ({\rm non-rotating}) \\ &= 0.42 \quad ({\rm maximal}) \end{aligned}$$

For $\epsilon = 0.1$, the accretion rate at the Eddington limit becomes,

$$\dot{M} \simeq 2 \left(\frac{M}{10^8 \ M_\odot} \right) \ M_\odot \ {\rm yr}^{-1}.$$

Super-Eddington accretion

May be possible to exceed the Eddington limit in magnetized atmospheres (Begelman 2001). Generally, the Eddington limit is a stronger constraint on the luminosity than on the accretion rate. At high $\dot{M}$, reach regime of photon-trapped accretion.

Consider a spherical accretion flow in free-fall. At radius r, radial velocity is,

$$v_r = -\sqrt{\frac{2GM}{r}}$$

density is given by,

$$4\pi r^2 \rho v_r = -\dot{M}$$

and free-fall timescale is,

$$t_{ff} = \frac{r}{|v_r|}.$$

Consider a photon diffusing out of the inflowing gas. After time t, number of scattering is,

$$N = \frac{tc}{l}$$

where $l = (\rho\kappa)^{-1}$ is the mean free path.

Distance travelled on a random walk is,

$$L = lN^{1/2}.$$

Photon will diffuse a distance r in a time,

$$t_{diff} = \frac{r^2}{c}\rho\kappa.$$

Photon diffuses *relative to the local fluid motion.* If $t_{diff} > t_{ff}$, then inflowing gas will drag photons inwards faster than they can diffuse outwards. Setting $t_{diff} = t_{ff}$ defines a *trapping radius*,

$$r_{trap} = \frac{\dot{M}c^2}{L_{Edd}}\left(\frac{GM}{c^2}\right)$$

...written in terms of the gravitational radius of the accreting object and assuming Thomson scattering opacity. Implies,

- Can have arbitrarily large $\dot{M}$, provided accretion energy is somehow lost (black hole: across horizon, neutron star: via neutrino emission).
- Escaping luminosity comparable to L_{Edd}.
- Relevant to neutron stars spiralling into stellar envelopes, possibly formation of massive black holes.

Opacity in stellar interiors

Numerical calculations are needed for accurate evaluation of the opacity. Standard references:

- *OPAL*: Rogers & Iglesias, 1992, ApJS, 79, 507. Iglesias & Rogers, 1996, ApJ, 464, 943.
- *Opacity project*: Seaton et al., 1994, MNRAS, 266, 805

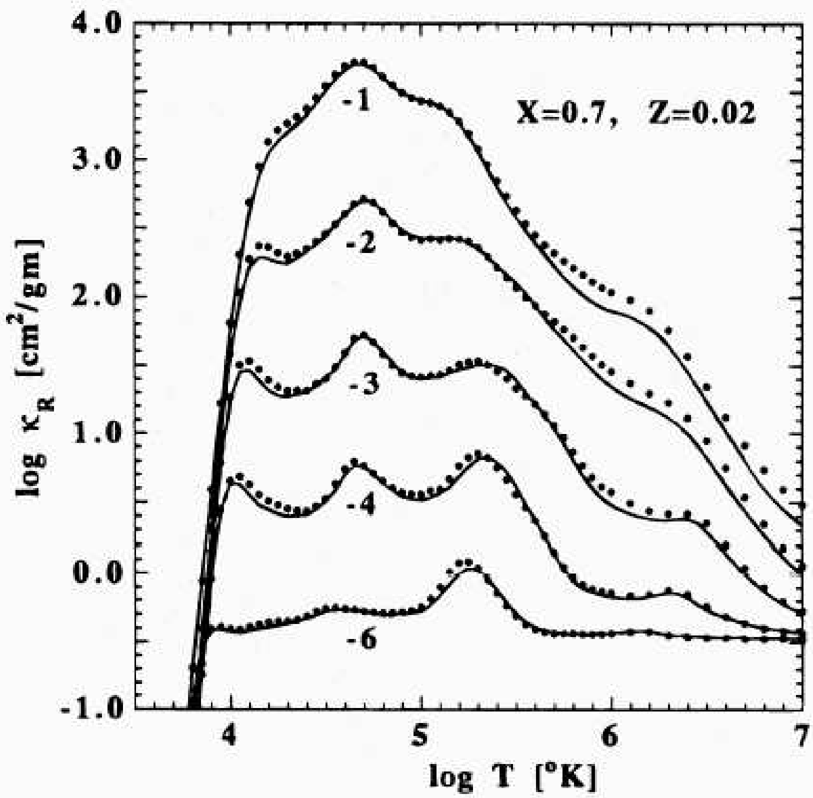
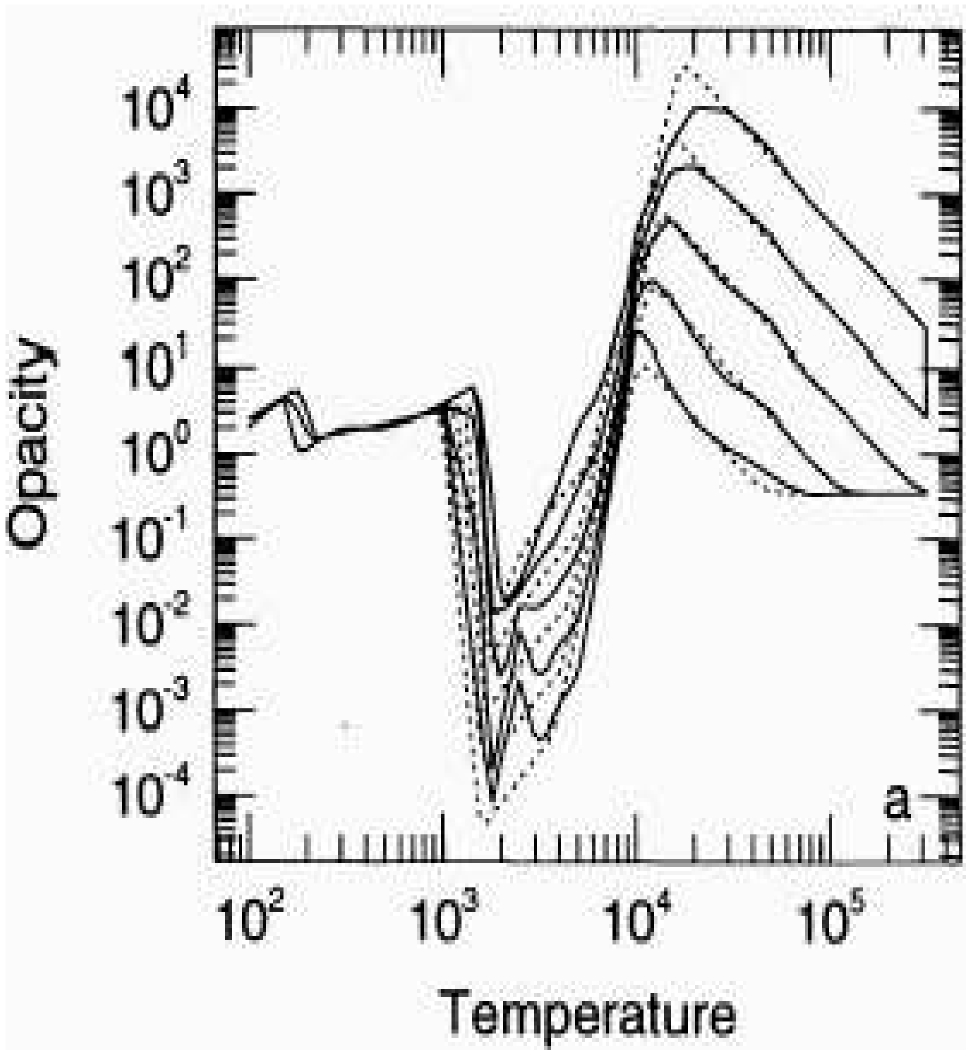


FIG. 7.—Comparison of updated OPAL (*dots*) and OP (*solid line*) Rosseland mean opacities at constant values of log *R* indicated in the figure where the metal distribution is from SYMP.

where $\log R \equiv \log(\rho/T_6^3)$ and $T_6 = 10^{-6}T$.

At lower temperatures (not normally required for stellar purposes), ices, dust and molecules need to be considered (see Alexander & Ferguson, 1994, ApJ, 437, 879).

At densities between 10^{-9} g cm^{-2} and 10^{-5} g cm^{-2}:



.

Peaks at a few $\times 10^4$ K due to the large contribution from atomic processes at this temperature.

Note: similar considerations lead to the cooling function (Dalgarno & McCray, 1972, ARA&A) peaking at $\sim 10^5$ K.

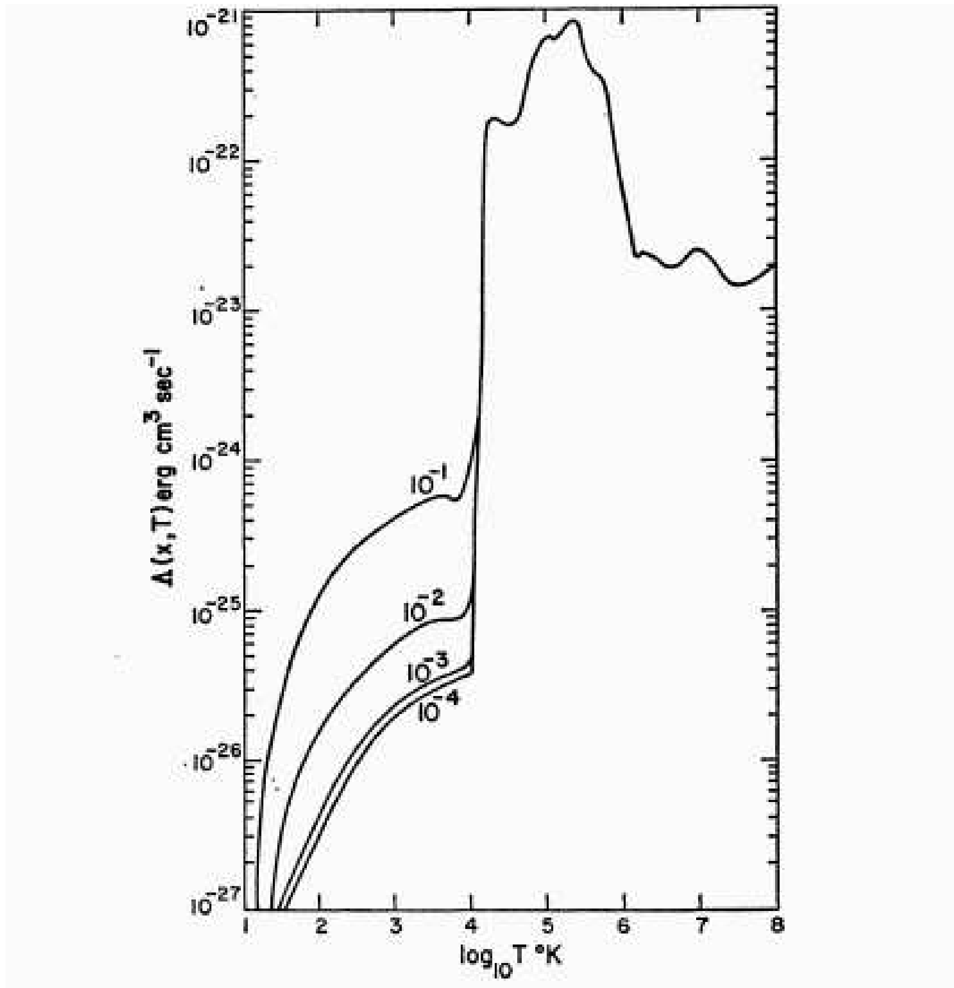


FIGURE 2. The interstellar cooling function $\Lambda(x, T)$ for various values of the fractional ionization x. The labels refer to the values of x.

(cooling rate per unit volume is $n^2\Lambda$).

Sources of opacity in stellar interiors

(1) Electron scattering:

If electrons are non-degenerate and non-relativistic, Thomson scattering with cross-section,

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = 0.6652 \times 10^{-24} \text{ cm}^2$$

provides a grey opacity – no frequency dependence. In practice, this is important at high T, where ionization is almost complete. For a hydrogen mass fraction X,

$$\kappa_{e-s} = 0.2\,(1+X) \;\text{ cm}^2 \text{ g}^{-1}.$$

No temperature or density dependence within the regime of applicability.

(2) Free-free absorption:

Absorption of a photon by a free electron in the Coulomb field of an ion.

See Hansen & Kawaler §4.4.2 for several approximate expressions for this opacity. Simplest, invalid for $T < 10^4$ K when the degree of ionization is too low, is,

$$\kappa_{f-f} \approx 4 \times 10^{22}\; (X+Y)(1+X)\rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1}$$

Inverse process is *bremsstrahlung*.

(3) Bound-free absorption or photoionization:

Can be approximated as:

$$\kappa_{b-f} \approx 4 \times 10^{25}\ Z(1+X)\rho T^{-3.5}\ \mathrm{cm}^2\ \mathrm{g}^{-1}.$$

Note: same dependence on density and temperature as for free-free opacity. Described as **Kramers' law**.

(4) Bound-bound opacity:

Usually smaller than the above processes, but not negligible.

(5) H^- opacity:

Free-free and bound-free opacities rise towards low T, until suppressed by lack of electrons. Steep dependence of the opacity with T at lower temperatures is due to opacity associated with the H^- ion (ionization potential 0.75 eV).

Very roughly,

$$\kappa_{H^-} \sim 2.5 \times 10^{-31}\ (Z/0.02)\rho^{1/2}T^9\ \mathrm{cm}^2\ \mathrm{g}^{-1}.$$

Most relevant to stellar atmospheres.