Convection: stability criteria

If the luminosity is transported by radiative diffusion, then the flux is,

$$F(r) = -\frac{4ac}{3\kappa_R \rho} T^3 \frac{dT}{dr}.$$

For illustrative purposes, assume that T^3/ρ is constant (as it would be for an $n = 3$ polytrope). Then,

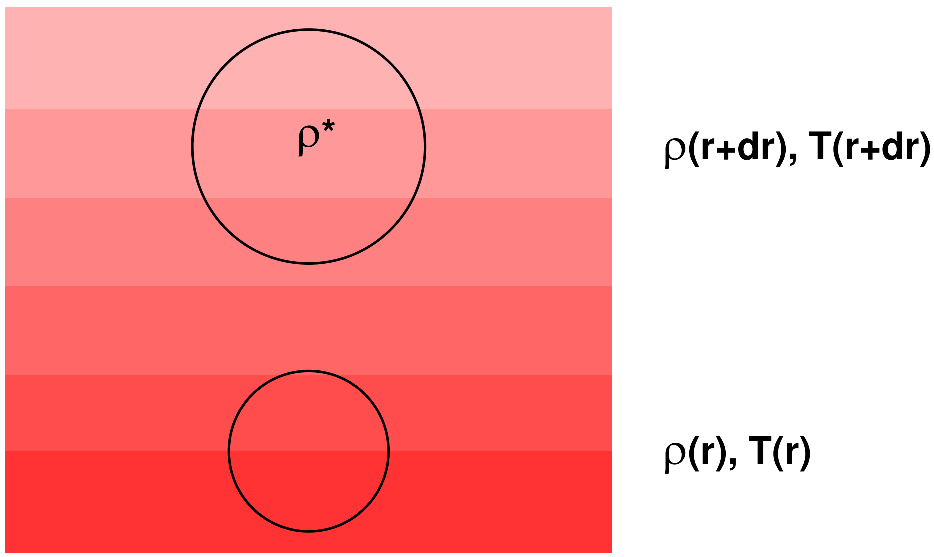
$$F(r) \propto \frac{1}{\kappa_R} \frac{dT}{dr}.$$

In a steady state, the energy flux at a given radius is set by the requirement that L_r equal the rate of interior energy generation. Thus, dT/dr will be large:

- If the luminosity is large.
- And / or if the opacity is large.

The gradient cannot become arbitrarily large, however. Instead, convection sets in.

Consider a medium of uniform composition with density and temperature profiles $\rho(r)$ and $T(r)$. Derive the Schwarzschild condition for convection by considering notional displacements of mass elements.



Displace a mass element dr *without exchanging heat with the environment*, i.e. adiabatically. What happens?

- Element expands to maintain pressure balance with the new environment.
- New density of the element ρ^* will *not* generally equal the ambient density at $r + dr$.
- Since the perturbation was adiabatic, for the element,

$$\frac{dP}{P} = -\Gamma_1 \frac{dV}{V} = \Gamma_1 \frac{d\rho}{\rho}.$$

New density of the element will be:

$$\rho^* = \rho(r) + d\rho = \rho(r) + \frac{1}{\Gamma_1}\frac{\rho}{P}\left(\frac{dP}{dr}\right)dr$$

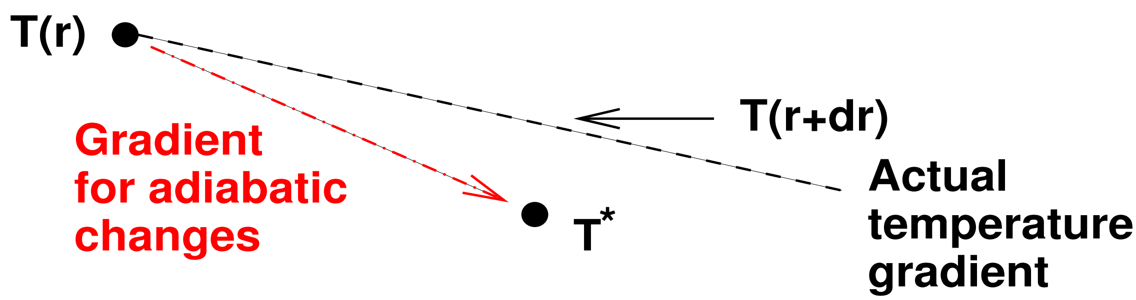
If,

- $\rho^* > \rho(r + dr)$ the displaced element will be denser than the surroundings and will settle back down $\rightarrow$ **stability**.
- $\rho^* < \rho(r + dr)$ buoyancy will cause the element to rise further $\rightarrow$ **instability**

This implies stability if we have,

$$\frac{1}{\Gamma_1}\frac{\rho}{P}\frac{dP}{dr} > \frac{d\rho}{dr}.$$

The same physics can be expressed in terms of the temperature gradient. In the stable case when $\rho^* > \rho(r+dr)$, pressure equilibrium requires that $T^* < T(r + dr)$. Graphically:



This is the **stable** case.

Stability condition can be written,

$$\left|\left(\frac{dT}{dr}\right)_{star}\right| < \left|\left(\frac{dT}{dr}\right)_{ad}\right|$$

where $(dT/dr)_{ad}$ is the *adiabatic temperature gradient*. Using the definition of the second adiabatic exponent,

$$\frac{dP}{P} + \frac{\Gamma_2}{1-\Gamma_2}\frac{dT}{T} = 0$$

the condition for stability becomes,

$$\left(\frac{dT}{dr}\right)_{star} > \left(1 - \frac{1}{\Gamma_2}\right)\frac{T}{P}\left(\frac{dP}{dr}\right)_{star}.$$

Note: both these gradients are negative, which makes this inequality potentially confusing. Physically: too rapid changes of T with r $\rightarrow$ convection.

We can convert this condition into a maximum luminosity that can be carried by radiation before convection sets in. Equation of radiative diffusion gives,

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa_R \rho}{T^3}\frac{L_r}{4\pi r^2}.$$

Stability requires,

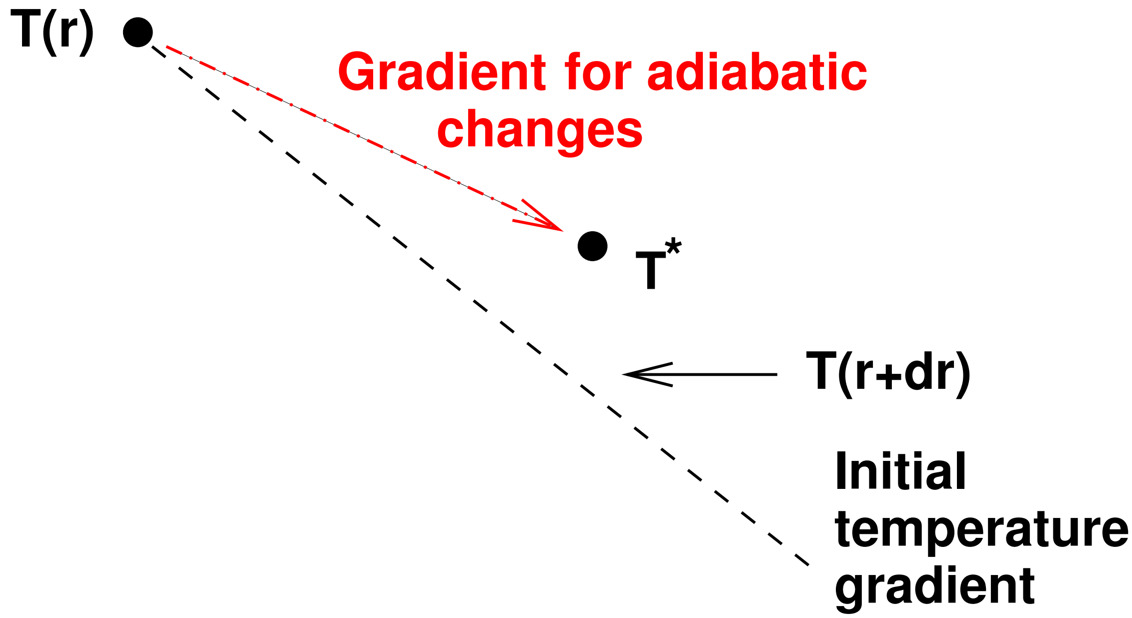
$$-\frac{3}{4ac}\frac{\kappa_R \rho}{T^3}\frac{L_r}{4\pi r^2} > \left(1 - \frac{1}{\Gamma_2}\right)\frac{T}{P}\left(\frac{dP}{dr}\right).$$

Now eliminate the pressure gradient using hydrostatic equilibrium,

$$L_r \leq \frac{16\pi acG}{3\kappa_R}\left(1 - \frac{1}{\Gamma_2}\right)\frac{T^4}{P}m(r).$$

If the luminosity exceeds this amount, convection will set in.

Graphically, easy to see that convective fluid motions will act as an additional source of energy transport:



In this, the unstable situation, the displaced element is **hotter** than its new surroundings. If we now relax the requirement of adiabaticity, radiation will leak out of the rising fluid element → an extra energy flux from hotter to cooler regions.

Obviously, we could equally well have considered *downward* adiabatic displacements.

What causes convective instability in some regions of a star but not in others? As an example, consider again a star with T^3/ρ equal to a constant. In the outer layers, $m(r) \simeq M$. The maximum luminosity then becomes,

$$L_{\max} \propto \frac{1}{\kappa_R}\left(1 - \frac{1}{\Gamma_2}\right).$$

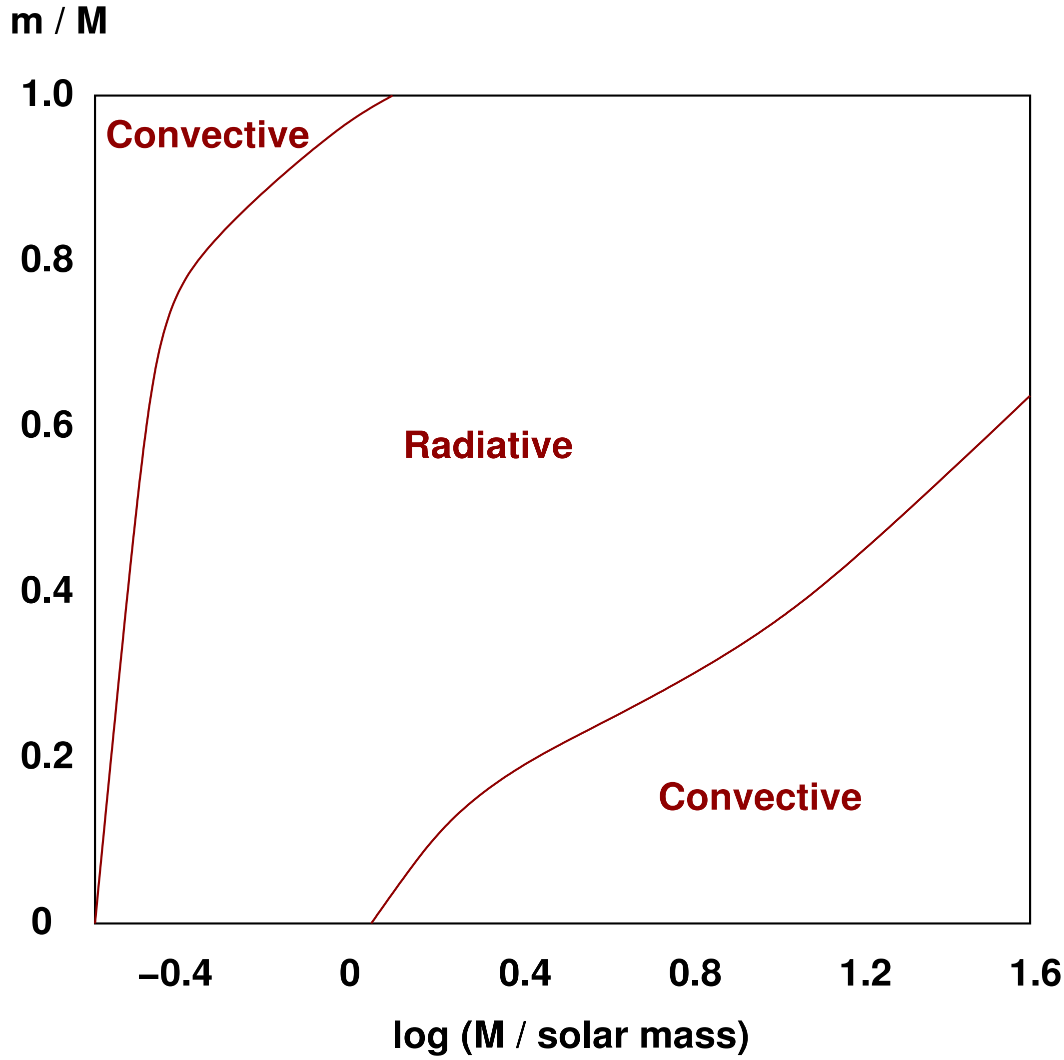
Vulnerable to convection if,

- $\Gamma_2 \to 1$. We have shown that the adiabatic exponents drop below 5/3 in ionization zones near the stellar surface.
- κ_R is large. This occurs when there is a large contribution to the opacity from atomic processes.

Both these arguments suggest that regions of the star where $T \sim 10^4 - 10^5$ K and ionization is occurring are liable to be convective. These are surface convection zones, and will not be present in massive stars with high effective temperatures.

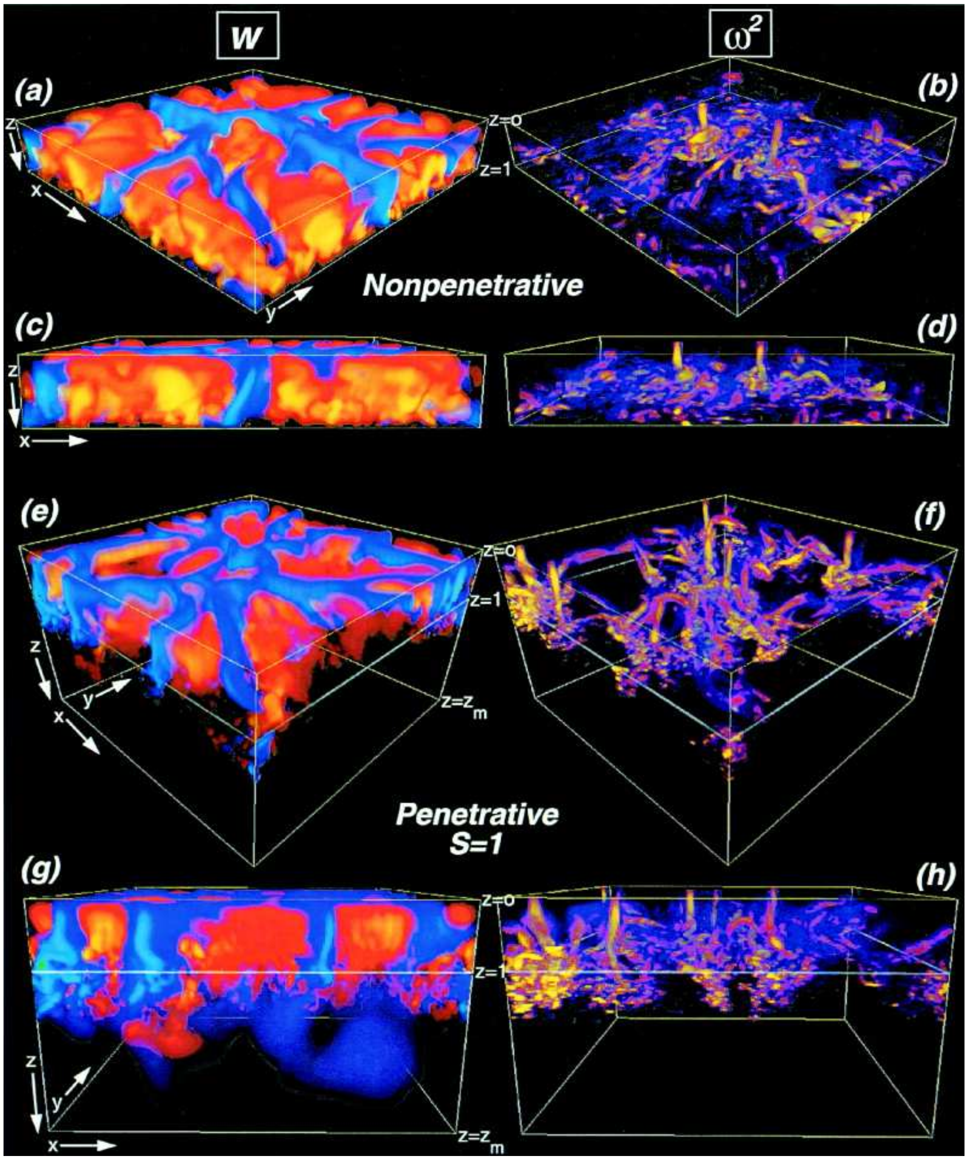
Convection zones can also occur in the core. A large luminosity at a small $m(r)$ is required. This will occur if the nuclear energy generation is a very very strong function of T – will show later that this occurs for CNO reactions in more massive stars.

Convective zones as a function of stellar mass for zero-age main sequence stars.



Is it a coincidence that surface convection zones end almost at the same mass as the core becomes convective?

What happens when convection sets in?



Enormously complicated fluid motions (e.g. Brummell et al. 2002)! Most stellar models use drastically simplified descriptions.