Conduction

Conduction is a diffusive energy transport process in which electrons rather than photons are the energy carriers. First justify neglect of conduction in ordinary (non-degenerate) stars.

For particles with mean free path l, and velocity v, random walk arguments show that the diffusive velocity across a scale R is,

$$v_{diff} \simeq \frac{l}{R} v.$$

The energy flux = energy density times the diffusive velocity. Comparing the energy flux from radiation to that from conduction,

$$\frac{F_{cond}}{F_{rad}} \approx \frac{P_g}{P_r} \times \frac{v_e}{c} \frac{l_e}{l_\gamma}.$$

Typical electron velocity is,

$$\frac{1}{2} m_e v_e^2 = \frac{3}{2} kT$$

$\rightarrow v_e \simeq 0.1$ c at $T = 1.5 \times 10^7$ K.

For electron mean free path, assume that collisions with ions are most efficient scattering mechanism. Mean free path is,

$$l_e = \frac{1}{n_I \sigma}$$

where n_I is the ion number density and $\sigma = \pi s^2$ is the cross-section. Estimate distance s over which scattering occurs by equating electron kinetic energy to electrostatic energy,

$$\frac{1}{2} m_e v_e^2 = \frac{Z e^2}{s}.$$

Substituting numbers for central Solar conditions ($T = 1.5 \times 10^7$ K, $\rho \simeq 10^2$ g cm^{-3}), obtain $l_e \sim 10^{-6}$ cm. Using $l_\gamma \sim 10^{-2}$, estimate,

$$\frac{F_{cond}}{F_{rad}} \approx 100 \times 0.1 \times 10^{-4} \sim 10^{-3}.$$

Conclude that conduction is not competitive with radiation in non-degenerate material, mainly because mean free path to scattering of charged particles is small.

Conduction in a degenerate gas

Conduction is enhanced in degenerate matter,

- High density means low energy quantum states are all filled. Electron velocity at top of Fermi sea is close to relativistic – $v_e \sim c$.
- Filled states suppress scattering. Only a small fraction of all electrons can scatter freely.

Can define a conductive opacity κ_{cond}. When radiation and conduction are involved,

$$\frac{1}{\kappa_{total}} = \frac{1}{\kappa_{rad}} + \frac{1}{\kappa_{cond}}$$

i.e. whichever opacity is lower determines the total. For almost complete degeneracy,

$$\kappa_{cond} \propto \rho^{-2} T^2$$

i.e. important at high density and low temperature.

Gravitational energy sources

In thermal balance,

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon.$$

In Lagrangian form,

$$\frac{dL_r}{dm} = \epsilon$$

where ϵ is the rate of local thermonuclear energy generation per unit mass (ie ϵ has units of erg g^{-1} s^{-1}). In a contracting (or expanding) star the heat content of mass elements will change,

$$\frac{dQ}{dt} = \epsilon - \frac{\partial L_r}{\partial m},$$

where Q is the heat content in erg g^{-1}. Define ϵ_{grav} such that,

$$\frac{\partial L_r}{\partial m} = \epsilon + \epsilon_{grav}$$

where,

$$\epsilon_{grav} = -\frac{dQ}{dt} = -\left[\frac{\partial E}{\partial t} + P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)\right].$$

This expression can be rewritten in the form,

$$\epsilon_{grav} = -\frac{P}{\rho(\Gamma_3 - 1)} \frac{\partial}{\partial t} \left[\ln \left(\frac{P}{\rho^{\Gamma_1}} \right) \right]$$

$\rightarrow$ departures from adiabaticity during contraction or expansion lead to energy release.

For a homologous expansion or contraction, the luminosity is related to the change in radius via,

$$L(t) = -q \frac{GM^2}{R^2} \left[\frac{\Gamma_1 - 4/3}{\Gamma_3 - 1} \right] \frac{dR}{dt},$$

where the gravitational energy,

$$\Omega = -q \frac{GM^2}{R}.$$

Identical to problem set 1 apart from clarification of Γ's.

Neutrino losses

Normally, L_r excludes energy flux in neutrinos. Strictly write,

$$\frac{\partial L_r}{\partial m} = \epsilon + \epsilon_{grav} - \epsilon_\nu,$$

where ϵ is the total nuclear energy generation rate and ϵ_ν the energy taken from the stellar material per unit mass per unit time in neutrinos.

Nuclear energy production

Suppose several lighter nuclei, with mass M_j, fuse to form a heavier nulceus of mass M_y. The energy liberated is,

$$E = \Delta M c^2$$

where the mass defect,

$$\Delta M = \sum_j M_j - M_y.$$

For ^{1}H (mass 1.0081 m_u) fusing to ^{4}He (mass 4.0039 m_u) the mass defect is around 0.7% of the original masses, or 26.5 MeV.

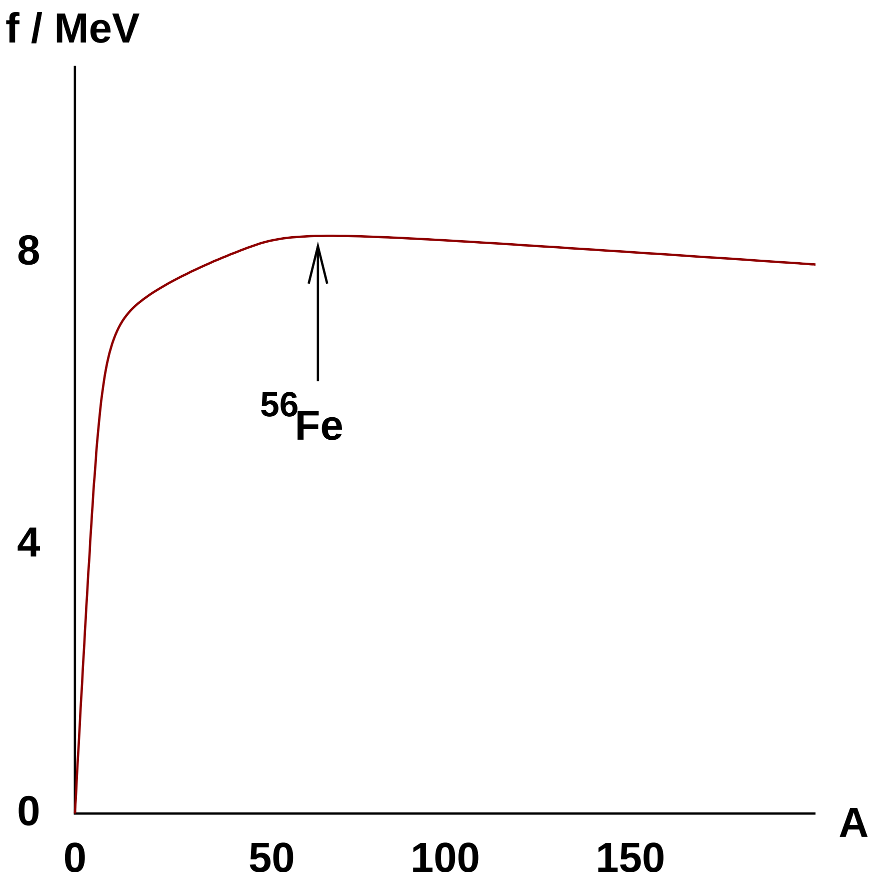
Define the binding energy of the nucleus as the energy required to separate the nucleons to infinity against the nuclear forces. For a nucleus with atomic mass number A and proton number Z,

$$E_B = \left[(A - Z)m_n + Z m_p - M_{nuc}\right] c^2,$$

where m_n and m_p are the neutron and proton masses. The binding energy per nucleon is,

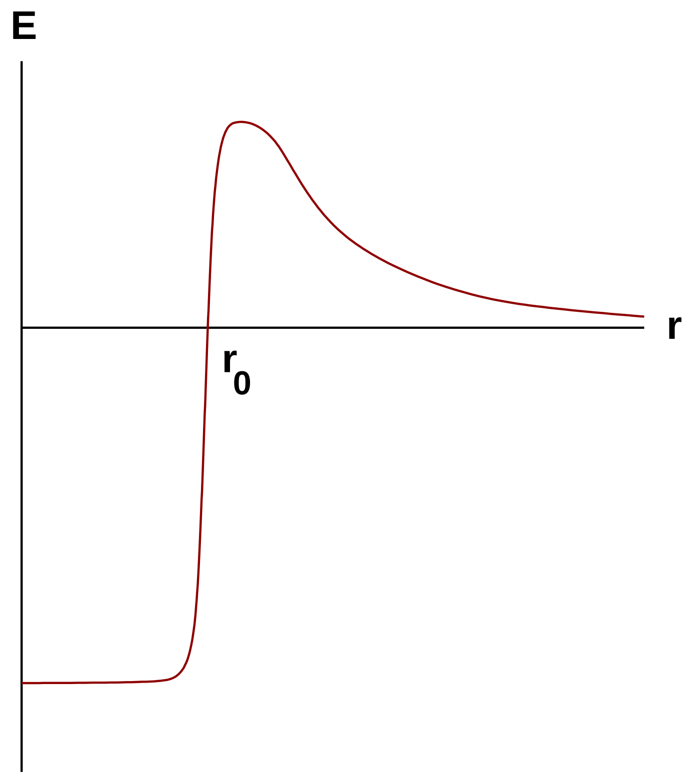
$$f = \frac{E_B}{A}.$$

Plot of f with A has the form (smoothed),



Most tightly bound nucleus is ^{56}Fe. Fusion of pure hydrogen to ^{56}Fe yields about 8.5 MeV per nucleon, the largest part of which (6.6 MeV) is already obtained in fusion to helium.

To fuse, nuclei have to surmount a Coulomb barrier,



Nuclear material has roughly constant density, so nuclear forces dominate within a radius,

$$r_0 \approx 1.44 \times 10^{-13}\ A^{1/3}\ \mathrm{cm}.$$

The height of the Coulomb barrier is thus,

$$E_C = \frac{Z_1 Z_2 e^2}{r_0} \sim Z_1 Z_2\ \mathrm{MeV}$$

for nuclei of charge Z_1, Z_2.

At $T = 10^7$ K, thermal energy kT of particles is of the order of 10^3 eV. Classically, therefore, there are *zero* particles in a thermal distribution with sufficent energy to fuse at this temperature.

Quantum mechanically, tunnelling probability is,

$$P_0 = p_o E^{-1/2} e^{-2\pi\eta}$$

where,

$$\eta = \left(\frac{m}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar E^{1/2}},$$

and m is the reduced mass. Note:

- This expression gives the dependence on the energy – p_0 varies depending on the nuclei involved.
- P_0 increases rapidly with E, and decreases with $Z_1 Z_2 \rightarrow$ lightest nuclei can generally fuse at lowest temperatures.
- Higher energies and temperatures are needed to fuse heavier nuclei $\rightarrow$ well-separated phases in which different elements burn during stellar evolution.