# Helium burning

First reaction of the helium burning chain is,

$$^4\text{He} +^4 \text{He} \rightarrow^8 \text{Be}.$$

This reaction is the inverse of the $^8$Be decay that terminates the PP-III chain, and is endothermic, absorbing 92 keV of energy.

Require roughly this much energy at the Gamov peak if $^8$Be is to exist in any significant numbers. Energy at the Gamov peak is,

$$\frac{E_0}{kT} \simeq 6.6 W^{1/3} \left( \frac{T}{10^7 \text{ K}} \right)^{-1/3}$$

where,

$$W = Z_j^2 Z_k^2 \frac{A_j A_k}{A_j + A_k}.$$

Setting $E_0$ equal to 92 keV, find $T \approx 1.15 \times 10^8$ K. This sets a rough lower limit for helium burning in conditions where electron screening can be neglected.

The equilibrium abundance of $^8$Be can be calculated using a nuclear version of the Saha equation. For the reaction,

$$\alpha + \alpha \leftrightarrow ^8 \mathrm{Be}$$

the Saha equation is,

$$\frac{n_\alpha^2}{n(^8\mathrm{Be})} = \left(\frac{\pi m_\alpha kT}{h^2}\right)^{3/2} e^{-Q/kT}$$

where,

- The Q-value of the reaction (-91.78 keV) replaces the ionization potential.
- The statistical weights of both an $\alpha$ particle and $^8$Be are unity.
- The electron mass $m_e$ has been replaced by the reduced mass $m_\alpha^2/m(^8\mathrm{Be}) \approx m_\alpha/2$.

Ignition of helium in lower mass stars starts at densities of $\rho \sim 10^6$ g cm$^{-3}$, implying $n_\alpha \approx 1.5 \times 10^{29}$ cm$^{-3}$ for pure helium.

At $10^8$ K, find,

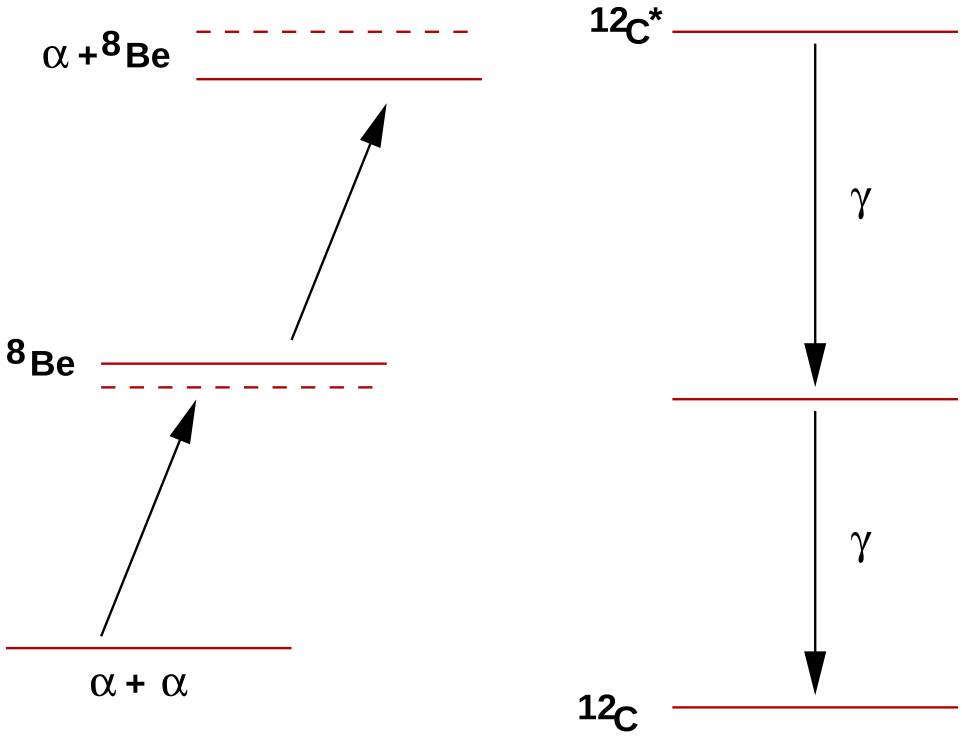
$$\frac{n(^8\mathrm{Be})}{n_\alpha} \simeq 7 \times 10^{-9}$$

i.e. a very low fraction.

The intermediate nucleus $^{8}$Be can then capture another $\alpha$ particle to form $^{12}$C,

$$^{8}\mathrm{Be} + \alpha \rightarrow ^{12}\mathrm{C} + \gamma + \gamma .$$

This reaction is exothermic and resonant, proceeding via an excited state $^{12}$C$^{*}$:



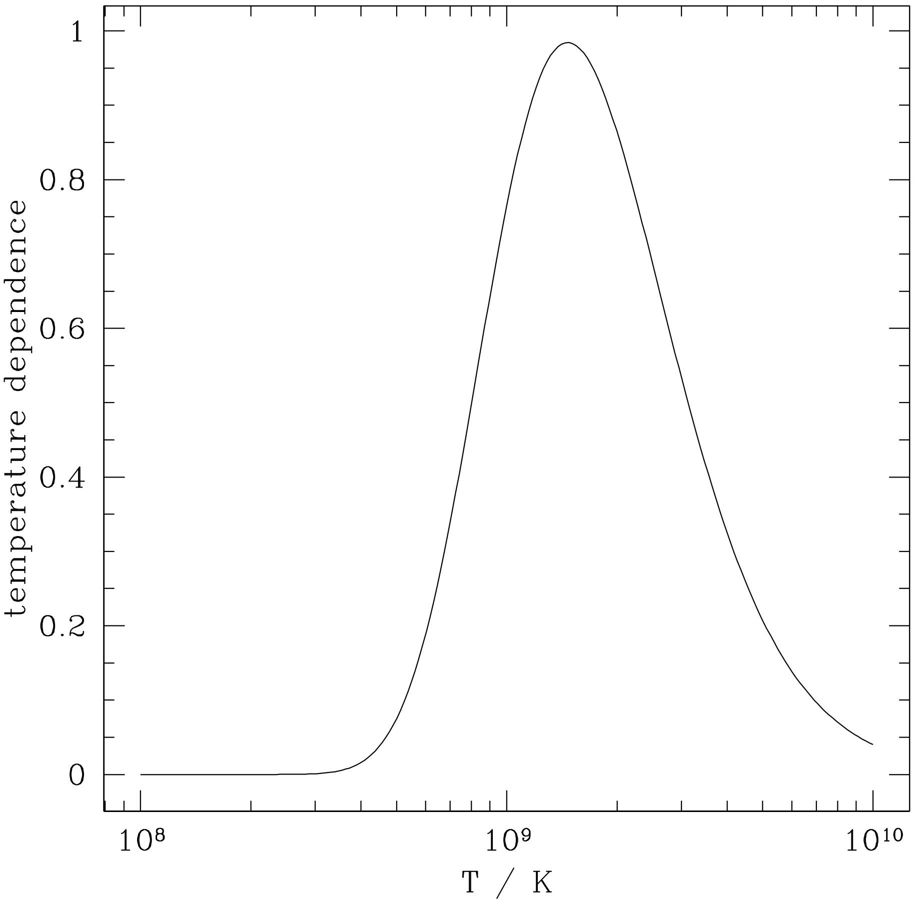
Again, the nuclear Saha equation can be used to find the concentration of $^{12}$C$^{*}$. Most $^{12}$C$^{*}$ decay straight back to $^{8}$Be plus an $\alpha$ particle, but some emit a photon to eventually form stable $^{12}$C.

Resulting energy generation rate from the triple $\alpha$ process is,

$$\epsilon_{3\alpha} = 5 \times 10^8 \, \rho^2 Y^3 \left( \frac{T}{10^9 \text{ K}} \right)^{-3} e^{-4.4027/T_9} \text{ erg g}^{-1} \text{ s}^{-1}$$

where $T_9$ is temperature in units of $10^9$ K and $Y$ is the mass fraction of helium.

Temperature dependent part of $\epsilon_{3\alpha}$ looks like,



The temperature sensitivity of the reaction is,

$$\nu_{3\alpha} \simeq \frac{4.4}{T_9} - 3$$

i.e. very high ($\nu_{3\alpha} \approx 40$) at temperatures of the order of $10^8$ K.

The effect of helium burning on the structure of the star depends upon the central conditions at the moment of ignition. These vary with the stellar mass:

- **Low mass stars** with $M < 0.4\ M_\odot$ develop a fully degenerate helium core *before* the temperature rises to the helium ignition threshold. The core cannot contract or heat up further. If the envelope is lost, the end state will be a helium white dwarf.
- **Higher mass stars** with $M > 1.5\ M_\odot$ ignite helium while the density is well below the degeneracy threshold ie $\rho \ll 10^6$ g cm$^{-3}$. No large structural change accompanies helium burning in such stars.
- **Intermediate mass stars** ignite helium under conditions of partial degeneracy. This allows a runaway nuclear reaction – the *helium flash*.

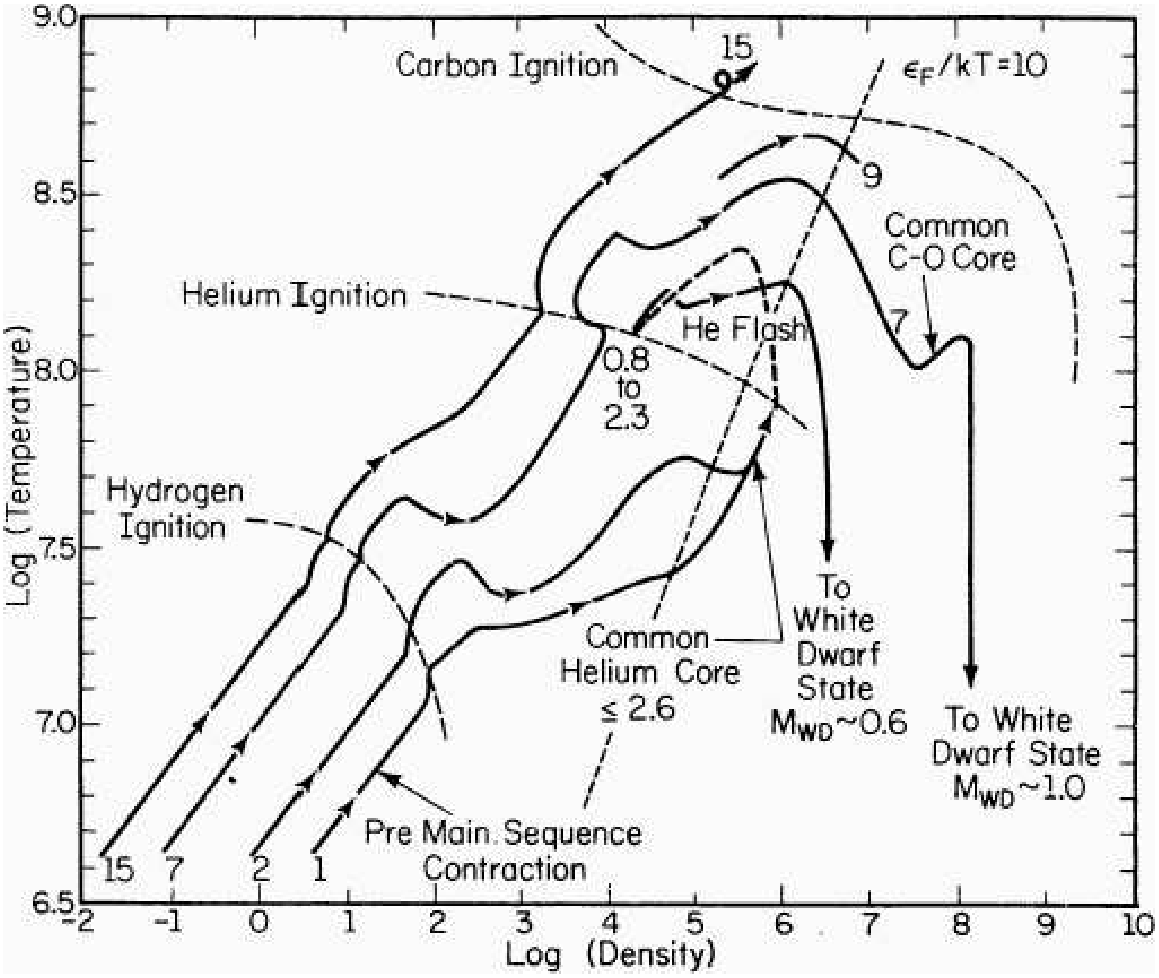
# The helium flash

Origin of the helium flash:

- Under degenerate conditions, the pressure is a function of density only, and not a function of temperature (this will only be approximately true under conditions of partial degeneracy).
- Upon helium ignition, the energy release leads to a temperature rise. But this does not lead to expansion and consequent reduction in the density and temperature as in a non-degenerate gas.
- The higher temperature increases the rate of the nuclear reaction further → runaway.

This continues until the temperature is high enough in the core that degeneracy is lifted. The core then expands rapidly and the structure adjusts until a new stable helium burning structure is attained, with a much lower core density.

Evolution of the core conditions (density and temperature) for stars of various masses is plotted by Iben (1985):



Note: different mass boundary between helium flash and stable burning regimes here is due to different metallicities and (probably) older stellar models.

Once enough $^{12}$C has been formed by the triple $\alpha$ reaction, further captures of $\alpha$ particles occur simultaneously with the carbon-forming reaction:

$$\begin{aligned} {}^{12}\mathrm{C} + \alpha &\rightarrow {}^{16}\mathrm{O} + \gamma \\ {}^{16}\mathrm{O} + \alpha &\rightarrow {}^{20}\mathrm{Ne} + \gamma \end{aligned}$$

In typical conditions, reactions beyond $^{20}$Ne are rare. The rates of these additional reactions are uncertain, but their yields need to be added to the yield of the triple $\alpha$ process.